

2018-2019 Guide

Eureka

Module 5: Numbers 10-20 and Counting to 100



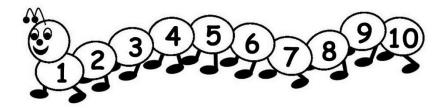
ORANGE PUBLIC SCHOOLS OFFICE OF CURRICULUM AND INSTRUCTION OFFICE OF MATHEMATICS

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Module 5 Performance Overview

- In Topic A, students start at the concrete level, counting 10 straws. Kindergarten students learn to comfortably talk about 10 ones, setting the foundation for the critical Grade 1 step of understanding 1 ten. They next separate 10 objects from within concrete and pictorial counts up to 20, analyzing the total as 10 ones and no ones or 10 ones and some ones (K.CC.1, K.NBT.1). They see two distinct sets which are then counted the Say Ten way: ten 1, ten 2, ten 3, ten 4, ten 5, ten 6, ten 7, ten 8, ten 9, 2 tens. Students hear the separation of the 10 ones and some ones as they count, solidifying their understanding as they also return to regular counting: eleven, twelve, thirteen, etc.
- In Topic B, the two distinct sets of ones are composed, or brought together, through the use of the Hide Zero cards (pictured below) and number bonds. Students represent the whole number numerically while continuing to separate the count of 10 ones from the count of the remaining ones with drawings and materials.
- In Topic C, the focus is now on the decomposition of the total teen quantity so that one part is ten ones. In this topic, unlike the previous two, the entire teen number is a whole quantity represented both concretely and pictorially in different configurations: towers or linear configurations, arrays (including the 10-frame or 5-groups), and circles.
- In Topic D, students extend their understanding of counting teen numbers to numbers from 21 to 100. They first count by tens both the Say Ten way—1 ten, 2 tens, 3 tens, 4 tens, etc.—and the regular way: twenty, thirty, forty, etc..
- In Topic E, students apply their skill with the decomposition and composition of teen numbers. The module closes with a culminating task, wherein students integrate all the methods they have used up until now to show decomposition.



Module 5: Numbers 10-20 and Counting to 100

Pacing:

April 29th – June 7th
30 Days

Topic	Les- son	Lesson Objective:		
Topic A:	Lesson 1	Count straws into piles of ten; count the piles as 10 ones.		
Count 10 and Some Ones	Lesson 2	Count 10 objects within counts of 10 to 20 objects, and describe as 10 ones and ones.		
	Lesson 3	Count and circle 10 objects within images of 10 to 20 objects, and describe as 10 ones and ones		
	Lesson 4	Count straws the Say Ten way to 19; make a pile for each ten		
	Lesson 5	Count straws the Say Ten way to 20; make a pile for each ten		
Topic B: Compose	Lesson 6	Model with objects and represent numbers 10 to 20 with place value or Hide Zero cards		
Numbers 11-20 from 10 Ones	Lesson 7	Model and write numbers 10 to 20 as number bonds.		
and some ones; Represent and	Lesson 8	Model teen numbers with materials from abstract to concrete.		
Write Teen Numbers	Lesson 9	Draw teen numbers from abstract to pictorial.		
Topic C:	Lesson 10	Build a Rekenrek to 20.		
Decompose Numbers 11-20	Lesson 11	Show, count, and write numbers 11 to 20 in tower configurations increasing by 1—a pattern of 1 larger.		
and Count to Answer "How	Lesson 12	Represent numbers 20 to 11 in tower configurations decreasing by 1—a pattern of 1 smaller.		
Many?" Questions in	Lesson 13	Show, count, and write to answer how many questions in linear and array configurations		
Varied Configurations	Lesson 14	Show, count, and write to answer how many questions with up to 20 objects in circular configurations.		
Mid- Module Assessment (Interview Style: 3 days)				
Topic D:	Lesson 15	May 17-21 st 2019 Count up and down by tens to 100 with Say Ten and regular counting.		
Extend the Say Ten and Regu-	Lesson 16	Count within tens by ones		

lar Count Sequence to	Lesson 17	Count across tens when counting by ones through 40.			
100	Lesson 18	Count across tens by ones to 100 with and without object.			
Topic E: Represent and	Lesson 20	Represent teen number compositions and decompositions as addition sentences.			
Apply Compositions and De-	Lesson 21	Represent teen number decompositions as 10 ones and some ones, and find a hidden part			
compositions of Teen Numbers	Lesson 22	Lesson 22: Decompose teen numbers as 10 ones and some ones; compare some ones to compare the teen numbers.			
	Lesson 23	Reason about and represent situations, decomposing teen numbers into 10 ones and some ones and composing 10 ones and some ones into a teen number			
	Lesson 24	Culminating Task—Represent teen number decompositions in various ways			
Enc	End-of- Module Assessment (Interview Style: 3 days)				
June 5-7th 2019					

NJSLS Standards:

Module 5: Numbers to 10-20 and Counting to 100



Write numbers from 0-20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects)

- Write the numerals 0-20
- Use the written numerals 0-20 to represent the amount within a set. **Example:** if the student has counted 9 objects, then the written numeral "9" is recorded.
- Students can record the quantity of a set by selecting a number card/tile (numeral recognition) or writing the numeral.
- Students can also create a set of objects based on the numeral presented. **Example:** if a student picks up the number card "13", the student then creates a pile of 13 counters. While children may experiment with writing numbers beyond 20, this standard places emphasis on numbers 0-20.
- Students should practice writing numerals with different kinesthetic modalities, such as sand or rice before they begin to write numbers on paper.

When counting objects, say the number of names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

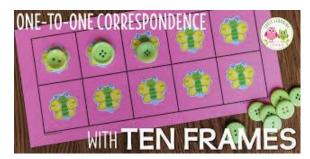
Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

K.CC.4a

K.CC.4b

K.CC.4c

Understand that each successive number name refers to a quantity that is one larger.



- Implement correct counting procedures by pointing to one object at a time (one-to-one correspondence)
- Use one counting word for every object (synchrony/ one-to-one tagging)
- Keep track of objects that have and have not been counted. This is the foundation of counting.

• Answer the question "How many are there?" by counting objects in a set and understanding that the last number stated when counting a set (...8, 9, 10) represents the total amount of objects:

Example: "There are 10 bears in this pile." (Cardinality)

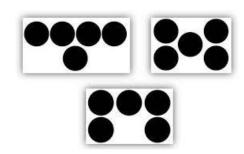
• Understanding that numbers build by exactly one each time and that they nest within each other by this amount.

Example: A set of three objects is nested within a set of 4 objects; within this same set of 4 objects is also a set of two objects and a set of one. Using this understanding, if a student has four objects and wants to have 5 objects, the student is able to add one more-knowing that four is within, or a sub-part of 5 (rather than removing all 4 objects and starting over to make a new set of 5).

- Students are asked to understand this concept with and without (0-20) objects. **Example:** After counting a set of 8 objects, students answer the question, "How many would there be if we added one more object?"; and answer a similar question when not using objects, by asking hypothetically, "What if we have 5 cubes and added one more. How many cubes would there be then?"
- Use five frames/ number paths to model linear representations of objects to help students begin to see patterns that make 5 with a variety of objects, such as buttons, counters, shells, coins, and dot cards. As students are ready, extend this work to 10 using the ten frame.

Count to tell the number of objects. count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

K.CC.5



- In order to answer "how many?" students need to keep track of objects when counting
- After numerous experiences with counting objects, along with the developmental understanding that a group of objects counted multiple times will remain the same amount, students recognize the need for keeping track in order to accurately determine "how many?"
- Depending on the amount of objects to be counted, and the students' confidence with counting a set of objects, students may move the objects as they count each, point to each object as counted, look without touching when counting, or use a combination of these strategies. It is important that children develop a strategy that makes sense to them based on the realization that keeping track is important in order to get an accurate count, as opposed to following a rule, such as "Line them all up before you count", in order to get the right answer.
- Some arrangements, such as a line or rectangular array, are easier for them to get the correct answer but may limit their flexibility with developing meaningful track-

- ing strategies.
- Providing multiple arrangements help children learn how to keep track. Since scattered arrangements are the most challenging for students, this standard specifies that students only count up to 10 objects in a scattered arrangement and count up to 20 objects in a line, rectangular array, or circle.
- Provide a variety of concrete experiences before students draw pictures.
- Students should count out a number of items using a variety of concrete objects, match numeral card with the number of items in a set, and count the number of items from a collection of items when given a written numeral.

K.NBT.1

Compose and decompose numbers from 11 to 19 into ten ones and some further ones e.g., by using objects or drawings and record each composition or decomposition by a drawing or equation (such as 18=10+8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones

- Students explore numbers 11-19 using representations, such as manipulatives or drawings. Keeping each count as a single unit, kindergarteners use 10 objects to represent "10" rather than creating a unit called a ten (unitizing) as indicated in the First Grade.
- Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.
- This Standard lays the foundation for further work with place value in Grade 1
 - Working with ten frames or rekenreks, which are powerful models, student may explore relationships between numbers
- Teen Number names may present a struggle for some students, and they will need many opportunities to compose groups of 10 with concrete materials

K.CC.1

Count to 100 by ones and by tens

- Students rote count by starting at one and counting to 100. When students count by tens they are only expected to master counting on the decade (0, 10, 20, 30, 40 ...). This objective does not require recognition of numerals. It is focused on the rote number sequence.
- It is normal for students to struggle with learning the sequence of number names, especially from 1-20, but our system of writing and naming numbers is simply a 'convention' that we can help our students learn.
- Encourage saying teen numbers the "Say Ten" Way in addition to conventional to connect the meaning

K.CC.2

Count forward beginning from a given number within the known sequence (instead of having to begin at 1)

- Students begin a rote forward counting sequence from a number other than 1. Thus, given the number 4, the student would count, "4, 5, 6, 7 ..." This objective does not require recognition of numerals. It is focused on the rote number sequence 0-100.
- Counting forward is more challenging for students than counting from 1 because they must up "in the middle (so to speak) of the counting sequence and recognize what comes next which requires a deeper understanding of our number system
- Counting forward lays the foundation for student s to "count on" to solve word problems in Grade 1.
- This standard is related to K.C.4c in which students recognize that each number represents "one more" than the number before it.

M: Major Content

S: Supporting Content

A: Additional Content

Teaching Representations/ Manipulatives/ Tools:

- 50 sticks or straws for each group of 2 students
- Student-made Rekenrek (pictured to the right): 10 red and 10 white pony beads, 1 cardboard strip, 2 elastics
- 1 egg carton per pair of students with 2 slots cut off to make a carton with 10 slots
- Hide Zero cards (called Place Value cards in later grades)

- 5-group cards
- Rekenrek (Slavonic abacus having beads with a color change at the five)
- Objects to put in the egg carton such as mandarin oranges, plastic eggs, or beans
- Single and double 10-frames
- Linking cubes: ideally 10 of two different colors per student
- Number bond template

Module 5 Assessment / Authentic Assessment Recommended Framework						
Assessment	Estimated Time	Format				
	Eureka Math					
Module 5: Nu	embers 10-20 and C	Counting to 10	0			
Optional Mid-Module Assessment (Interview Style)	K.CC.1 K.CC.3 K.CC.4bc K.CC.5 K.NBT.1	1 Block	Individual or Small Group with Teacher			
Optional End of Module Assessment (Interview Style)	K.CC.1 K.CC.2 K.CC.5 K.NBT.1	1 Block	Individual or Small Group with Teacher			

Number Talks Cheat Sheet

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?

Kindergarten Ideal Math Block

Getting Ready: Whole Group

Anchor Task: Math In Focus Learn

7-10 min.

Fluency: Whole Group

Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group

Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

50-60 min.

Concept Development: Individual/partner/whole

Instruction & Strategic Problem Set Questions

Student Debrief: Independent Exit Ticket

CENTERS/ LEARNING STATIONS:

Pairs / Small Group/ Individual

DIFFERENTIATED activities designed to **RETEACH**, **REMEDIATE**, **ENRICH** student's understanding of concepts.

M: Meet with the teacher A:
Application/
Problem Solving

T: Technology H: Hands on Activities

SUMMARY: Whole Group

Lesson Closure: Student Reflection; Real Life Connections to Concept

Up to 5 min.

15-20 min.

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting: Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

Terminology

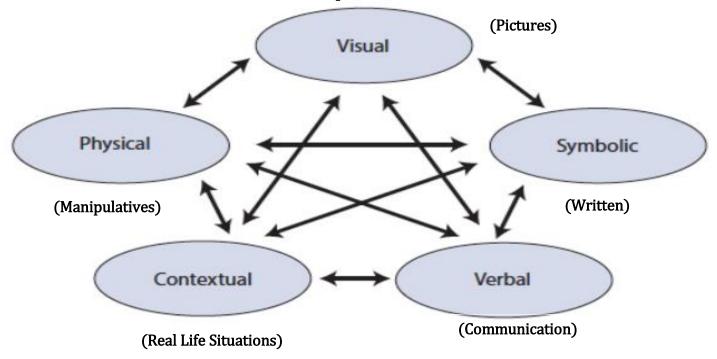
- 10 and __
- 10 ones and some ones
- 10 plus
- Hide Zero cards (called Place Value cards in later grades, pictured to the right)
- Regular counting by ones from 11 to 20 (eleven, twelve, thirteen, etc.)
- Regular counting by tens to 100 (e.g., ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, one hundred)
- Say Ten counting by tens to 100 (e.g., 1 ten, 2 tens, 3 tens, 4 tens, 5 tens, 6 tens, 7 tens, 8 tens, 9 tens, 10 tens)
- Teen numbers

PARCC Assessment Evidence/Clarification Statements					
ccss	Evidence Statement	Clarification	Math Prac- tices		
K.CC.B.5	Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.	i) Tasks may have a context. ii) Tasks should include a range of counting exercises to answer "how many" objects in different arrangements progressing to the more difficult action of counting out a given number of objects. iii) Interviews (individual or small group) should target students' abilities to meet this evidence statement.	MP.7		

Student Name:	Task:	School:	Teacher:	Date:
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	STUDENT FRIENDLY RUBRIC				
"I CAN"	a start 1	getting there 2	that's it 3	WOW! 4	SCORE
Understand	I need help.	I need some help.	I do not need help.	I can help a class- mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my thinking.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: "Doing Stage": Physical manipulation of objects to solve math problems.

Pictorial: "Seeing Stage": Use of imaged to represent objects when solving math problems.

Abstract: "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over.... And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing? **WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

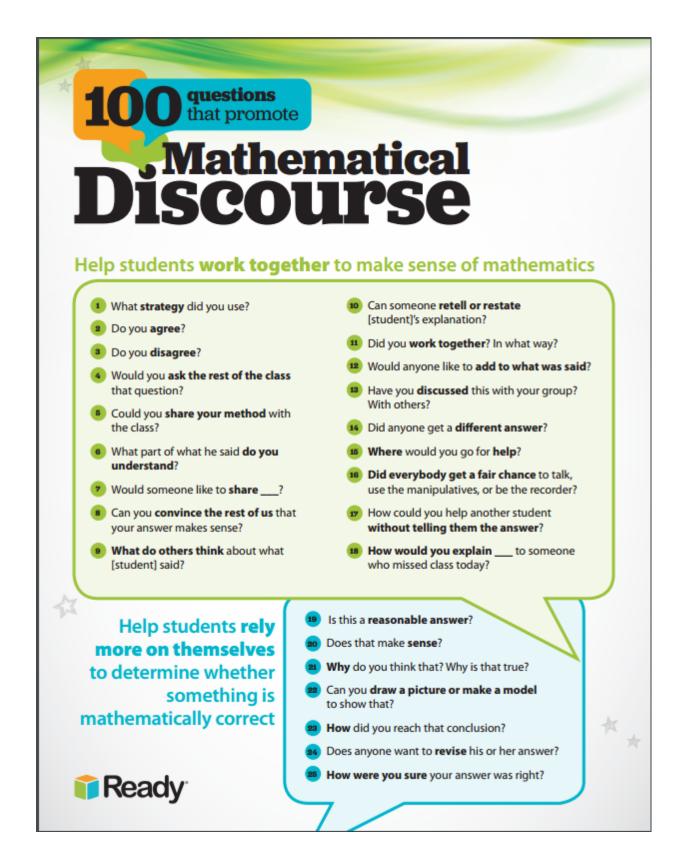
Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



Help students learn to reason mathematically

- How did you begin to think about this problem?
- What is another way you could solve this problem?
- How could you prove ____
- Can you explain how your answer is different from or the same as [student]'s answer?
- Let's break the problem into parts. What would the parts be?
- Can you explain this part more specifically?
- Does that always work?
- Can you think of a case where that wouldn't work?
- 34 How did you organize your information? Your thinking?

Help students with problem comprehension

Help students evaluate their own processes and engage in productive peer interaction

- What do you need to do next?
- 36 What have you accomplished?
- What are your strengths and weaknesses?
- Was your group participation appropriate and helpful?
 - What is this problem about? What can you tell me about it?
 - O Do you need to define or set limits for the problem?
 - How would you interpret that?
 - Could you reword that in simpler terms?
 - 43 Is there something that can be eliminated or that is missing?
 - Could you explain what the problem is asking?
 - What assumptions do you have to make?
 - What do you know about this part?
 - Which words were most important? Why?



100 Questions That Promote Mathematical Discourse 2



- What would happen if ___?
- Do you see a pattern?
- What are some possibilities here?
- Where could you find the information you need?
- How would you check your steps or your answer?
- What did not work?
- How is your solution method the same as or different from [student]'s method?
- Other than retracing your steps, how can you determine if your answers are appropriate?
- 66 How did you organize the information? Do you have a record?
- How could you solve this using tables, lists, pictures, diagrams, etc.?
- What have you tried? What steps did you take?
- 69 How would it look if you used this model or these materials?

- How would you draw a diagram or make a sketch to solve the problem?
- 61 Is there another possible answer? If so, explain.
- Is there another way to solve the problem?
- Is there another model you could use to solve the problem?
- Is there anything you've overlooked?
- How did you think about the problem?
- 66 What was your estimate or prediction?
- How confident are you in your answer?
- What else would you like to know?
- What do you think comes next?
- Is the solution reasonable, considering the context?
- Did you have a system? Explain it.
- Did you have a strategy? Explain it.
- Did you have a design? Explain it.





100 Questions That Promote Mathematical Discourse 3

Help students learn to connect mathematics, its ideas, and its application

- What is the relationship between ____
- Have we ever solved a problem like this before?
- What uses of mathematics did you find in the newspaper last night?
- What is the same?
- What is different?
- Did you use skills or build on concepts that were not necessarily mathematical?
- Which skills or concepts did you use?
- What ideas have we explored before that were useful in solving this problem?

- Is there a pattern?
- Where else would this strategy be useful?
- How does this relate to ?
- Is there a general rule?
- Is there a real-life situation where this could be used?
- How would your method work with other problems?
- What other problem does this seem to lead to?
 - Have you tried making a guess?
 - What else have you tried?
 - Would another method work as well or better?
 - 92 Is there another way to draw, explain, or say that?
 - Give me another related problem. Is there an easier problem?
 - How would you explain what you know right now?

Help students persevere

- What was one thing you learned (or two, or more)?
- Did you notice any patterns? If so, describe them.
- What mathematics topics were used in this investigation?
- What were the mathematical ideas in this problem?
- What is mathematically different about these two situations?
- What are the variables in this problem? What stays constant?

Help students focus on the mathematics from activities

Ready

100 Questions That Promote Mathematical Discourse 4

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the <u>mind</u> with the low-level details required, allowing it to become an automatic response pattern or <u>habit</u>. It is usually the result of learning, repetition, and practice.

K-2 Math Fact Fluency Expectation

K.OA.5 Add and Subtract within 5.

1.0A.6 Add and Subtract within 10.

2.0A.2 Add and Subtract within 20.

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.0A.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
- decomposing a number leading to a ten (e.g., 13 4 = 13 3 1 = 10 1 = 9);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- o place value,
- o properties of operations, and/or
- o the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

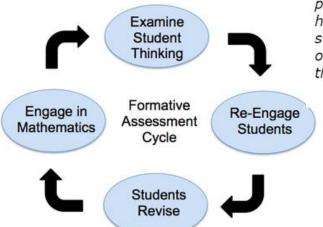
Mathematical Proficiency

To be mathematically proficient, a student must have:

- <u>Conceptual understanding</u>: comprehension of mathematical concepts, operations, and relations;
- <u>Procedural fluency</u>: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- <u>Strategic competence</u>: ability to formulate, represent, and solve mathematical problems;
- <u>Adaptive reasoning</u>: capacity for logical thought, reflection, explanation, and justification;
- <u>Productive disposition</u>: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

- 1. I can solve problems without giving up.
- 2. I can think about numbers in many ways.
- 3. I can explain my thinking and try to understand others.
- 4. I can show my work in many ways.
- 5. I can use math tools and tell why I choose them.
- 6. I can work carefully and check my work.
- 7. I can use what I know to solve new problems.
- 8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Make sense of problems and persevere in solving them

In Kindergarten, students learn that doing math involves solving problems and discussing how they solved them. Students will begin to explain the meaning of a problem, and look for ways to solve it. Kindergarteners will learn how to use objects and pictures to help them understand and solve problems. They will begin to check their thinking when the teacher asks them how they got their answer, and if the answer makes sense. When working in small groups or with a partner they will listen to the strategies of the group and will try different approaches.

Reason abstractly and quantitatively

Mathematically proficient students in Kindergarten make sense of quantities and the relationships while solving tasks. This involves two processes- decontextualizing and contextualizing.

In Kindergarten, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 7 children on the playground and some children go line up. If there are 4 children still playing, how many children lined up?" Kindergarten students are expected to translate that situation into the equation: 7-4 =___, and then solve the task.

Students also contextualize situations during the problem solving process. For example, while solving the task above, students refer to the context of the task to determine that they need to subtract 4 since the number of children on the playground is the total number of students except for the 4 that are still playing. Abstract reasoning also occurs when students measure and compare the lengths of objects.

Construct viable arguments and critique the reasoning of others

Mathematically proficient students in Kindergarten accurately use mathematical terms to construct arguments and engage in discussions about problem solving strategies. For example, while solving the task, "There are 8 books on the shelf. If you take some books off the shelf and there are now 3 left, how many books did you take off the shelf?" students will solve the task, and then be able to construct an accurate argument about why they subtracted 3 form 8 rather than adding 8 and 3. Further, Kindergarten students are expected to examine a variety of problem solving strategies and begin to recognize the reasonableness of them, as well as similarities and differences among them.

Model with mathematics

4

Mathematically proficient students in Kindergarten model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context.

Kindergarten students rely on concrete manipulatives and pictorial representations while solving tasks, but the expectation is that they will also write an equation to model problem situations.

For example, while solving the task "there are 7 bananas on the counter. If you eat 3 bananas, how many are left?" Kindergarten students are expected to write the equation 7-3=4.

Likewise, Kindergarten students are expected to create an appropriate problem situation from an equation.

For example, students are expected to orally tell a story problem for the equation 4+5=9.

Use appropriate tools strategically

Mathematically proficient students in Kindergarten have access to and use tools appropriately. These tools may include counters, place value (base ten) blocks, hundreds number boards, number lines, and concrete geometric shapes (e.g., pattern blocks, 3-d solids). Students should also have experiences with educational technologies, such as calculators, virtual manipulatives, and mathematical games that support conceptual understanding.

During classroom instruction, students should have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while solving the task "There are 4 dogs in the park. If 3 more dogs show up, how many dogs are they?"

Kindergarten students are expected to explain why they used specific mathematical tools."

Attend to precision

Mathematically proficient students in Kindergarten are precise in their communication, calculations, and measurements. In all mathematical tasks, students in Kindergarten describe their actions and strategies clearly, using gradelevel appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions.

For example, while measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions.

Look for and make use of structure

Mathematically proficient students in Kindergarten carefully look for patterns and structures in the number system and other areas of mathematics. While solving addition problems, students begin to recognize the commutative property, in that 1+4=5, and 4+1=5.

While decomposing teen numbers, students realize that every number between 11 and 19, can be decomposed into 10 and some leftovers, such as 12 = 10+2, 13 = 10+3, etc.

Further, Kindergarten students make use of structures of mathematical tasks when they begin to work with subtraction as missing addend problems, such as 5 - 1 = can be written as 1 + = 5 and can be thought of as how much more do I need to add to 1 to get to 5?

Look for and express regularity in repeated reasoning

Mathematically proficient students in Kindergarten begin to look for regularity in problem structures when solving mathematical tasks.

Likewise, students begin composing and decomposing numbers in different ways.

For example, in the task "There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?"

Kindergarten students are expected to realize that the 8 crayons could include 4 of each color (4+4=8), 5 of one color and 3 of another (5+3=8), etc.

For each solution, students repeated engage in the process of finding two numbers that can be joined to equal 8.

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Pract	ices for Orchestrating Productive Mathematics Discussions
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task?
	How do you respond to the work that students are likely to produce?
	Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson.
	Students working in pairs or groups
	Listening to and making note of what students are discussing and the strategies they are using
	Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.
	Selection of children is guided by the mathematical goal for the lesson
4. Sequencing	What order will the solutions be shared with the class?
	Sequence depends largely on the teacher's goals for a lesson
5. Connecting	Asking the questions that will make the mathematics explicit and understandable.
	Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

ath Workstation:	 	 Time:
JSLS.:		
ojective(s): By the end o	e to:	
•	 	
•	 	
sk(s):		
•	 	
•	 	
•		
•	 	
•it Ticket:		
•it Ticket:		

MATH WORKSTATION SCHEDULE

TT7 1	C
Week	ot.
VVCCK	OI.

DAY	Technology	Problem Solving Lab	Fluency	Math	Small Group Instruc-
	Lab		Lab	Journal	tion
Mon.					
	Group	Group	Group	Group	BASED
Tues.					ON CURRENT
	Group	Group	Group	Group	OBSERVATIONAL
Wed.					DATA
	Group	Group	Group	Group	
Thurs.					
	Group	Group	Group	Group	
Fri.					
	Group	Group	Group	Group	

INSTRUCTIONAL GROUPING

	INSTRUCTION	III ONO	31110
	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

Kindergarten PLD Rubric

60	t It	Kindergarten PLD Rubric				
Evidence shows that the studen		Not There Yet Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a fail-				
cept or big math idea.	t essentially has the target con-	ure to engage in the task.				
PLD Level 5: 100%	PLD Level 4: 89%	PLD Level 3: 79%	PLD Level 2: 69%	PLD Level 1: 59%		
	Strong Command	Moderate Command	Partial Command	Little Command		
Distinguished command Student work shows distin-		Student work shows moderate		Student work shows little un-		
guished levels of understand-	Student work shows strong levels of understanding of the	levels of understanding of the	Student work shows partial understanding of the mathe-	derstanding of the mathemat-		
ing of the mathematics.	mathematics.	mathematics.	matics.	ics.		
ing of the mathematics.	mathematics.	mathematics.	matics.	ics.		
Student constructs and com-	Student constructs and com-	Student constructs and com -	Student constructs and com-	Student attempts to constructs		
municates a complete re-	municates a complete re-	municates a complete response	municates an incomplete re-	and communicates a response		
sponse based on explana-	sponse based on explana-	based on explana-	sponse based on student's at-	using the:		
tions/reasoning using the:	tions/reasoning using the:	tions/reasoning using the:	tempts of explanations/ rea-	• Tools:		
• Tools:	• Tools:	• Tools:	soning using the:	o Manipulatives		
Manipulatives	Manipulatives	Manipulatives	• Tools:	o Five Frame		
o Five Frame	o Five Frame	o Five Frame	 Manipulatives 	o Ten Frame		
o Ten Frame	o Ten Frame	o Ten Frame	o Five Frame	 Number Line 		
 Number Line 	 Number Line 	 Number Line 	o Ten Frame	o Part-Part-Whole		
o Part-Part-Whole	o Part-Part-Whole	o Part-Part-Whole	 Number Line 	Model		
Model	Model	Model	o Part-Part-Whole	Strategies:		
Strategies:	Strategies:	Strategies:	Model	o Drawings		
 Drawings 	 Drawings 	o Drawings	Strategies:	 Counting All 		
 Counting All 	 Counting All 	 Counting All 	 Drawings 	o Count On/Back		
 Count On/Back 	o Count On/Back	o Count On/Back	 Counting All 	 Skip Counting 		
 Skip Counting 	 Skip Counting 	 Skip Counting 	Count On/Back	Making Ten		
 Making Ten 	o Making Ten	o Making Ten	o Skip Counting	o Decomposing		
 Decomposing 	o Decomposing	o Decomposing	o Making Ten	Number		
Number	Number	Number	 Decomposing 	Precise use of math vo-		
Precise use of math vo-	Precise use of math vo-	Precise use of math vo-	Number	cabulary		
cabulary	cabulary	cabulary	Precise use of math vo-	D : 1 1 1 1 1 1		
Response includes an efficient	Decrease in classical and a	Demonstrated as a least 11	cabulary	Response includes limited evi-		
and logical progression of	Response includes a logical	Response includes a logical but	Dogwana in alteda an in acces	dence of the progression of		
mathematical reasoning and	progression of mathematical	incomplete progression of mathematical reasoning and	Response includes an incom- plete or illogical progression of	mathematical reasoning and		
understanding.	reasoning and understanding.	understanding.	mathematical reasoning and	understanding.		
		Contains minor errors .	understanding.			
5 points	4 points	3 points	2 points	1 point		
J politis	T politics	J politics	2 politis	1 point		

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form	School:	Teacher:	Date:	
Assessment:		NJSLS:		
		- ·,- — · ·		
GROUPS (STUDENT INITIALS)	SUPPORT PLAN		PROGRESS	
MASTERED (86% - 100%) (PLD 4/5):				
DEVELOPING (67% - 85%) (PLD 3):				
MCEGUDE (540) (50) (DI D 0)				
INSECURE (51%-65%) (PLD 2):				
BEGINNING (0%-50%) (PLD 1):				

MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual",
 "Partner/Group", and "Individual w/Opportunity for Student Interviews1.
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended*.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)².
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio.

Resources

Number Book Assessment Link: http://investigations.terc.edu/

Model Curriculum- http://www.nj.gov/education/modelcurriculum/

Georgia Department of Education: Games to be played at centers with a partner or small group. http://ccgpsmathematicsk-5.wikispaces.com/Kindergarten

Engage NY: *For additional resources to be used during centers or homework.

https://www.engageny.org/sites/default/files/resource/attachments/math-gk-m1-full-module.pdf

Add/ Subtract Situation Types: Darker Shading indicates Kindergarten expectations https://achievethecore.org/content/upload/Add%20Subtract%20Situation%20Types.pdf

Math in Focus PD Videos: https://www-

 $\underline{\texttt{k6.thinkcentral.com/content/hsp/math/hspmath/common/mif_pd_vid/9780547760346_te/index.}$ $\underline{\texttt{html}}$

Number Talk/string videos https://hcpss.instructure.com/courses/124/pages/routines:

Suggested Literature

Fish Eyes by, Lois Ehlert

Ten Little Puppies by, Elena Vazquez

Zin! Zin! Zin! A Violin! by, Lloyd Moss

My Granny Went to the Market by, Stella Blackstone and Christopher Corr

Anno's Counting Book by, Mitsumasa Anno

Chicka, Chicka, 1,2,3 by, Bill Martin Jr.; Michael Sampson; Lois Ehlert

How Dinosaurs Count to 10 by Jane Yolen and Mark Teague

10 Little Rubber Ducks by Eric Carle

Ten Black Dots by Donald Crews

Mouse Count by Ellen Stoll Walsh

Count! by Denise Fleming

21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see 21st Century Career Ready Practices .

References

"Eureka Math" Great Minds. 2018 < https://greatminds.org/account/products